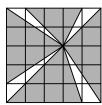
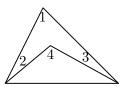
San Diego Math League High School Division, Round 1b November 14, 2009

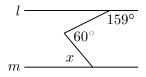
- 1. On Monday, a box of donuts is marked down 10%. On Tuesday, it is marked down another 20%. The total overall discount is equal to x%. What is x?
- 2. Find k if the points (2, -3), (4, 3), and (5, k/2) lie on a line.
- 3. In the diagram below, find the ratio of the shaded area to the unshaded area.



4. In the figure below, what is the measure of angle 4 if the measures of the angles 1, 2, and 3 are 76 , 27° , and 17 , respectively?



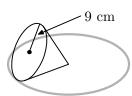
- 5. In a store, there are 7 cases containing a total of 128 apples altogether. Find the largest number N so that one can be sure that some case contains at least N apples.
- 6. In the diagram below, lines l and m are parallel. Find the measure of angle x.



- 7. The function f(x) satisfies f(x+y) = f(x) + f(y) + 6xy + 1 and f(x) = f(-x) for all integers x and y. Find f(3).
- 8. In a bag of three balls, there are exactly two red balls. If Alex randomly draws two balls without replacement, the probability of drawing the two red balls is 1/3. However, before Alex draws his two balls, additional balls are added to the bag. The probability of drawing two red balls without replacement is still 1/3. What is the least number of balls that could be in the bag (before Alex draws two balls) after the additional balls have been added?

 $\frac{1}{2}$ revolutions of the cone

to form one complete circumference of the circle described by the pale line. What is the volume of the cone?



10. How many positive divisors of 2009¹⁰ are perfect squares?

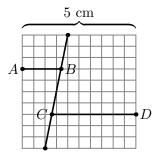
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- 11. Ten balls numbered 1 to 10 are in a bag. Marx reaches into the bag and randomly draws one of the balls. Then James reaches into the bag and randomly draws a different ball. What is the probability that the sum of the two numbers on the balls is even?
- 12. Find the infinite sum

$$1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 2} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 2 \cdot 3 \cdot 4} + \cdots,$$

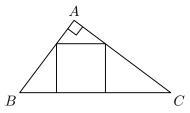
where each term is obtained by multiplying the previous term by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$, successively in that order.

- 13. Find the number of ways of arranging the letters A, B, C, D, E, and F, so that A comes before B, C comes before D, and E comes before F.
- 14. Two real numbers x and y satisfy $x^2 + \frac{1}{x^2} = 2009^2 + \frac{1}{2009^2}$ and $y^2 + \frac{1}{y^2} = 2010^2 + \frac{1}{2010^2}$. What is the largest possible positive difference between x and y?
- 15. Segment AB in the diagram below is 1.7 cm long. Find the length of CD.



the probability that the resulting integer will be divisible by 11?

17. Right triangle ABC has one leg of length 6 cm, one leg of length 8 cm, and a right angle at A. A square has one side on the hypotenuse of the triangle ABC, and has a vertex on each of the legs of the triangle ABC. What is the length of one side of the square?



- 18. Let ABCD be a trapezoid, with $AB \parallel CD$. Diagonals AC and BD intersect at E. The areas of triangles ABE and CDE are 27 and 48, respectively. Find the area of trapezoid ABCD.
- 19. Let a and b be real numbers such that a + b = 2 and $a^4 + b^4 = 56$. Find $a^2 + b^2$.
- 20. A great circle is a circle drawn on a sphere whose center is also the center of the sphere. Eight great circles are drawn on a sphere, so that no three great circles pass through the same point. Find the number of regions these eight great circles divide the surface of the sphere into.